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Pion Compton Scattering, e^+e^- Annihilation into Hadrons, and Electromagnetic Form Factors*

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A model of pion Compton scattering is formulated on the basis of Regge poles and scale invariance. The structure functions for $e^+ + e^- \to \pi^\pm$ + hadrons are derived from those for electroproduction by using crossing symmetry, and the cross section for e^+e^- annihilation is calculated by assuming factorization of the Regge residues. The predicted cross section is consistent with recent storage-ring experimental results. The proton and pion electromagnetic form factors are then calculated from finite-energy sum rules and good agreement with the data is obtained. It is found that the e^+e^- annihilation cross section is not inconsistent with a pion form factor which decreases rapidly with increasing momentum transfer squared.

I. INTRODUCTION

Much interest is focused on the e^+e^- annihilation experiment being performed at the Frascati[1] storage ring, and on future storage-ring experiments planned elsewhere. Preliminary data on multiparticle production in e^+e^- inelastic scattering have already been obtained and the results indicate that the production cross section falls off more slowly than anticipated from calculations based on the vector-dominance model.[2] A previous calculation[3] of the cross section for $e^+ + e^- \to p$ + hadrons, based on a Regge-pole model with scale invariance at large values of q^2 (the photon mass squared), predicted that σ behaved as $\sigma \approx 1/q^2$ for $q^2 \to \infty$.

In the following, we shall develop a model of pion Compton scattering by extending our previous work.[3] The residues of the exchanged Regge poles P and P' are calculated by assuming that the residues factorize into products of particle-particle Regge-pole couplings so that they can be calculated from the residues of other known processes. By means of unitarity, we can then calculate the structure functions for electroproduction, and by using s-u crossing symmetry obtain the structure functions for the e^+e^- pair-annihilation process by an analytic continuation, since this process is dominated by the same t-channel exchanges as the electroproduction process. By adjusting one free parameter, we then predict the cross section for $e^+ + e^- \to \pi^\pm$ + hadrons and find agreement with the preliminary Frascati data.[1]

The structure functions for the processes $e^- + p \to e^-$ + hadrons and $e^- + \pi^\pm \to \pi^\pm$ + hadrons are then used in a finite-energy sum rule (FESR) which connects the elastic electromagnetic form factors of the proton and pion to an integral over these structure functions in the "scale-invariance" region. Good agreement with the data for both the proton and pion form factors is obtained in the region of energy for which the Regge-pole model is applicable. The pion form factor is predicted to fall off as $(-q^2)^{-3/2}$ asymptotically, where $-q^2$ is the momentum transfer squared; this shows that a slowly decreasing cross section for e^+e^- annihilation is consistent with a decreasing pion form factor, which corresponds to a composite pion.

The paper has six sections. In Sec. II, we present the kinematics and amplitudes used to describe pion Compton scattering. In Sec. III, we define the annihilation structure functions. In Sec. IV, we give our model for the Compton amplitude, and in Sec. V, we describe the predictions of the model for the annihilation process and for the pion and proton form factors, and compare these to the experimental data.

II. PION COMPTON SCATTERING

In Fig. 1, the kinematics of the process $\pi^\pm + \gamma \to \pi^\pm + \gamma$ for photons of mass $\sqrt{q^2}$ are described. The forward amplitude for this process is given by

$$T^\pi(\nu, q^2) = 4\pi\alpha i \epsilon_2^{\mu *} \epsilon_1^\nu \times \int d^4x\, e^{iq\cdot x} \langle p|\theta(x_0)[J_\mu^{\text{em}}(x), J_\nu^{\text{em}}(0)]|p\rangle, \tag{2.1}$$

where ϵ_1^μ and ϵ_2^ν are the polarization vectors of the incoming and outgoing photons, respectively. Moreover, $\nu = p\cdot q/\mu$ is the laboratory energy of the incoming photon, μ is the pion mass, J_μ^{em} is the hadron electromagnetic current, and $\alpha = e^2/4\pi$. By charge-conjugation invariance, the amplitudes for $\pi^+\gamma$ and $\pi^-\gamma$ elastic scattering are equal.

Let us define the tensor

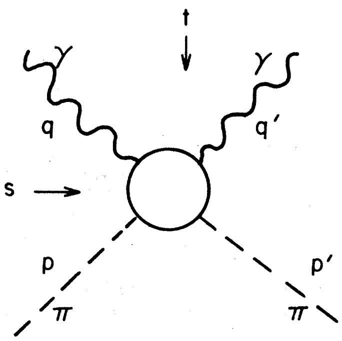

FIG. 1. Definition of kinematical quantities for $\gamma\pi^{\pm}$ elastic scattering.

$$\frac{1}{\mu}T^{*\pi}_{\mu\nu}=\frac{iE_\pi}{\mu^2}\int d^4x\, e^{iq\cdot x}\theta(x_0)\langle p|[J^{\rm em}_\mu(x),J^{\rm em}_\nu(0)]|p\rangle$$
$$+\text{ polynomials in } q. \tag{2.2}$$

This tensor satisfies the condition of gauge invariance

$$q^\mu T^{*\pi}_{\mu\nu}=q^\nu T^{*\pi}_{\mu\nu}=0\,. \tag{2.3}$$

We can write the covariant tensor $T^{*\pi}_{\mu\nu}$ in terms of two Lorentz-invariant amplitudes T^π_1 and T^π_2 as follows:

$$\frac{1}{\mu}T^{*\pi}_{\mu\nu}=\frac{1}{\mu^2}\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)T^\pi_2(\nu,q^2)$$
$$-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)T^\pi_1(\nu,q^2). \tag{2.4}$$

Forward $\pi^+\gamma$ scattering can be described in terms of a Sommerfeld-Watson representation of the t-channel process $\gamma+\gamma\to\pi^++\pi^-$ continued from the region $t>4\mu^2$, $q^2>0$ to the region $t\leqslant 0$ and $q^2<0$ with $\nu=p\cdot q/\mu$ physical, and where $t=(p-p')^2=(q-q')^2$ is the momentum transfer squared. The representation for T_1 is given for large ν and $t=0$ by the real analytic function

$$T^\pi_1(\nu,q^2)=-\pi\sum_i(2\alpha_i+1)\frac{\beta^\pi_i(q^2,\alpha_i)}{\sin\pi\alpha_i}[\nu^{\alpha_i}+(-\nu)^{\alpha_i}], \tag{2.5}$$

where we have summed over the Regge poles with intercepts α_i. The discontinuity of T^π_1 above the cut for positive ν is given by

$$\frac{1}{\pi}\,\mathrm{Im}T^\pi_1(\nu,q^2)=\sum_i(2\alpha_i+1)\beta^\pi_i(q^2,\alpha_i)\nu^{\alpha_i}\,. \tag{2.6}$$

If we define $\bar{T}^\pi_1(\nu,q^2)$ as the physical amplitude for the u-channel reaction, i.e., $\pi^-\gamma$ scattering, for negative ν, we have, taking the discontinuity below the cut and using crossing symmetry,

$$\frac{1}{\pi}\mathrm{Im}\bar{T}^\pi_1(\nu,q^2)=\sum_i(2\alpha_i+1)\beta^\pi_i(\nu,q^2)(-\nu)^{\alpha_i}\,. \tag{2.7}$$

It follows that the discontinuity of $\bar{T}^\pi_1(\nu,q^2)$ satisfies

$$\mathrm{Im}T^\pi_1(\nu,q^2)=\mathrm{Im}\bar{T}^\pi_1(-\nu,q^2). \tag{2.8}$$

In the t channel only $C=+1$ Regge poles are exchanged, and because G parity forbids A_2 exchange, only P and P' exchanges need be considered as dominant with $\alpha_P=1$ and $\alpha_{P'}\approx\frac{1}{2}$.

III. ELECTROPRODUCTION AND e^+e^- PAIR ANNIHILATION

We shall now discuss inelastic electron scattering $e^-+\pi^\pm\to e^-+$ hadrons and electron-positron pair annihilation $e^++e^-\to\pi^\pm+$ hadrons. In Fig. 2, we describe the inelastic scattering. In the laboratory frame

$$\nu=p\cdot q/\mu=E-E'$$

is the photon energy, where E and E' are the electron initial and final energies, respectively. The invariant mass of the hadrons produced is

$$\sqrt{s}\equiv W=(\mu^2+q^2+2\mu\nu)^{1/2} \tag{3.1}$$

and in the present metric ($p^2=p_0^{\,2}-\vec{p}^{\,2}=\mu^2$)

$$q^2=-4EE'\sin^2(\tfrac{1}{2}\theta), \tag{3.2}$$

where θ is the laboratory scattering angle of the electron. The threshold for elastic scattering is $W=\mu$ or $q^2=-2\mu\nu$.

By using unitarity, we can relate the forward scattering amplitude for massive photons scattering off pions to the structure functions for deep-inelastic electroproduction

$$\mathrm{Im}T^{*\pi}_{\mu\nu}=\pi W^\pi_{\mu\nu}, \tag{3.3}$$

where

$$W^\pi_{\mu\nu}=\frac{E_\pi}{\mu}\sum_n\langle p|J^{\rm em}_\mu(0)|n\rangle\langle n|J^{\rm em}_\nu(0)|p\rangle$$
$$\times(2\pi)^4\delta(q+p-p_n). \tag{3.4}$$

The tensor $W^\pi_{\mu\nu}$ can be written as

$$\frac{1}{\mu}W^\pi_{\mu\nu}=\frac{1}{\mu^2}\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)W^\pi_2(\nu,q^2)$$
$$-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)W^\pi_1(\nu,q^2), \tag{3.5}$$

where W^π_i are the structure functions satisfying

$$\mathrm{Im}T^\pi_i(\nu,q^2)=\pi W^\pi_i(\nu,q^2). \tag{3.6}$$

We shall assume that the "scale-invariance" limit, in the sense of Bjorken,[4] holds in a nontrivial way for the pion structure functions W^π_i. This assumption is motivated by an extension of Bjorken's derivation of "scale invariance" of the structure functions describing the inelastic

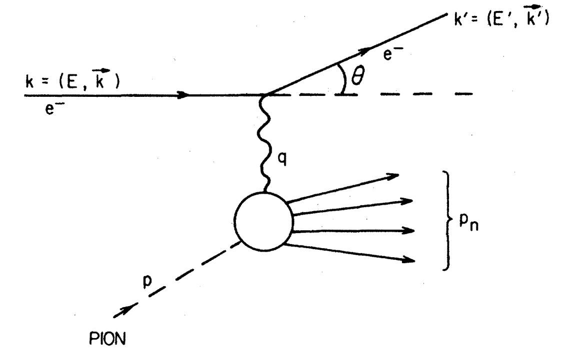

FIG. 2. Kinematics of the reaction $e^- + \pi^\pm \to e^- +$ hadrons.

process $e^- + p \to e^- +$ hadrons to the case of electrons scattering off *any* hadron. We cannot, however, *prove* that the limit of "scale invariance" is a nontrivial one. Specifically, we assume that in the limit $-q^2 \to \infty$, $\nu \to \infty$, $\omega \equiv -2\mu\nu/q^2$ fixed, the W_i^π have the nonzero limits

$$\lim_{\nu\to\infty;\ \omega \text{ fixed}} 2\mu W_1^\pi(\nu, q^2) = F_1^\pi(\omega),$$
$$\lim_{\nu\to\infty;\ \omega \text{ fixed}} \nu W_2^\pi(\nu, q^2) = F_2^\pi(\omega). \tag{3.7}$$

The annihilation structure functions $\overline{W}_{\mu\nu}^\pi$ are defined by

$$\frac{1}{\mu}\overline{W}_{\mu\nu}^\pi = \frac{E_\pi}{\mu^2}\sum_n \langle 0|J_\mu^{\text{em}}(0)|n\rangle\langle n|J_\nu^{\text{em}}(0)|0\rangle \times (2\pi)^4\delta^4(q-p-p_n)$$
$$= \frac{1}{\mu^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\overline{W}_2^\pi(\nu, q^2) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\overline{W}_1^\pi(\nu, q^2), \tag{3.8}$$

where the kinematics are described in Fig. 3; note that $q^2 > 0$ and $-1 < \omega < 2\mu/-q$; also the state n includes at least one pion.

We define $\tilde{W}_i^\pi(\nu, q^2)$ as the continuation of $W_i^\pi(\nu, q^2)$ to negative ν; from (2.8) we see

$$\tilde{W}_i^\pi(-\nu, q^2) = W_i^\pi(\nu, q^2), \quad q^2 < 0. \tag{3.9}$$

The annihilation process involves a further continuation to $q^2 > 0$. This continuation cannot be shown generally to be valid, as the amplitudes are not analytic in q^2; it is, however, true in specific models, of which ours is an example. If it is possible to perform the continuation, then by applying the reduction formalism to the pion in (3.8) and (3.4), we can show

$$\overline{W}_{\mu\nu}^\pi(q, -p) = \tilde{W}_{\mu\nu}^\pi(q, p). \tag{3.10}$$

Hence, using (3.9), we have

$$\overline{W}_i^\pi(\nu, q^2) = W_i^\pi(-\nu, q^2), \tag{3.11}$$

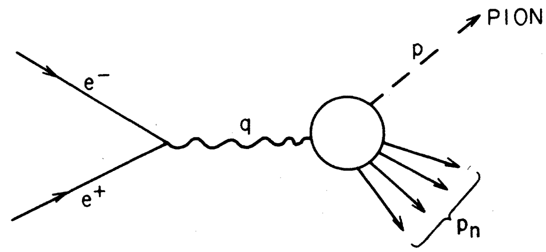

FIG. 3. Kinematics of the annihilation process $e^+ + e^- \to \pi^\pm +$ hadrons.

where $W_i^\pi(-\nu, q^2)$ has been continued in q^2 to $q^2 > 0$.

We note that in the case of γp scattering, (3.11) becomes

$$\overline{W}_i^p(\nu, q^2) = -W_i^p(-\nu, q^2) \tag{3.12}$$

because of Fermi-Dirac statistics.

The annihilation cross section in the barycentric frame of the colliding-beam experiment is given by

$$\frac{d^2\sigma}{d\Omega dE_\pi} = \frac{2\alpha^2}{q^4}\frac{\mu^2\nu}{\sqrt{q^2}}\left(1 - \frac{q^2}{\nu^2}\right)^{1/2} \times\left[2\overline{W}_1^\pi(\nu, q^2) + \frac{2\mu\nu}{q^2}\left(1 - \frac{q^2}{\nu^2}\right)\frac{\nu\overline{W}_2^\pi(\nu, q^2)}{2\mu}\sin^2\theta\right], \tag{3.13}$$

where E_π is the energy of the final pion, and θ is the angle of the pion momentum with respect to the axis determined by the incident e^+ and e^- beams.

If we assume that the transverse- and longitudinal-photon matrix elements are equal, then

$$2\mu W_1^\pi = \omega(\nu W_2^\pi) \tag{3.14}$$

and

$$2\mu\overline{W}_1^\pi = \omega(\nu\overline{W}_2^\pi). \tag{3.15}$$

The cross section (3.13) then becomes

$$\frac{d^2\sigma}{d\Omega dE_\pi} = \frac{2\alpha^2}{q^4}\frac{\mu\nu}{\sqrt{q^2}}\left(1 - \frac{q^2}{\nu^2}\right)^{1/2} \times 2\mu\overline{W}_1^\pi(\nu, q^2)\left[1 - \frac{1}{2}\left(1 - \frac{q^2}{\nu^2}\right)\sin^2\theta\right]. \tag{3.16}$$

IV. MODEL FOR PION COMPTON AMPLITUDE AND e^+-e^- ANNIHILATION

Our model takes the form

$$T_1^\pi(\nu, q^2, t) = -\pi\left(\frac{\omega^2 - 1}{\omega^2 + \eta^2}\right)^2 \times\sum_i \beta_i^\pi\left(\frac{2\mu\nu}{-q^2 + \lambda^2}\right)^{\alpha_i(t)}\xi_i(t)\frac{\Gamma(\alpha_i(0))}{(2\mu)^{\alpha_i(0)}}, \tag{4.1}$$

where for all ν the signature factor is

$$\xi(t)=\frac{1+e^{-i\pi\alpha_i(t)}}{\Gamma(\alpha_i(t))\sin\pi\alpha_i(t)}\,. \qquad (4.2)$$

This choice of signature factor corresponds to approaching the cut from above for $\nu>0$ and from below for $\nu<0$, giving in both cases the imaginary part of the physical amplitude for the s-channel $(\pi^+\gamma)$ and the u-channel $(\pi^-\gamma)$ processes, respectively. The factor $\Gamma(\alpha_i(t))$ cancels the pole at $\alpha_i=0$, and η and λ are constants. The index i runs over the dominant t-channel Regge poles, i.e., the P and the P'.

We now determine the residues of the P and the P' using factorization. We write the asymptotic total cross section for particles A and B as a sum over t-channel Regge poles α_i; i.e.,

$$\sigma_T(AB)=\sum_i\beta_i(AB)s^{\alpha_i(0)-1}. \qquad (4.3)$$

Factorization tells us that the residues $\beta_i(AB)$ factorize into two couplings:

$$\beta_i(AB)=\gamma_i(AA)\gamma_i(BB).$$

Thus $(i=P,P')$

$$\begin{aligned}\beta_i(\gamma N)&=\gamma_i(\gamma\gamma)\gamma_i(NN),\\ \beta_i(\gamma\pi)&=\gamma_i(\gamma\gamma)\gamma_i(\pi\pi),\\ \beta_i(\pi N)&=\gamma_i(NN)\gamma_i(\pi\pi).\end{aligned} \qquad (4.4)$$

Hence it follows that

$$\beta_i(\gamma\pi)=\frac{\beta_i(\gamma N)\beta_i(\pi N)}{\beta_i(NN)}\,. \qquad (4.5)$$

The residues $\beta_i^{\pi,p}$ used in the model for $T_1^{\pi,p}$ are related to the residues $\beta_i(\gamma\pi)$ and $\beta_i(\gamma N)$ as follows $(q^2=0)$:

$$\begin{aligned}\beta_i(\gamma\pi)&=4\pi^2\alpha\frac{\beta_i^\pi}{\lambda^2}\left(\frac{1}{2\mu\lambda^2}\right)^{\alpha_i(0)-1},\\ \beta_i(\gamma p,\gamma n)&=4\pi^2\alpha\frac{\beta_i^{p,n}}{m_0^2}\left(\frac{1}{2Mm_0^2}\right)^{\alpha_i(0)-1}.\end{aligned} \qquad (4.6)$$

The values for β_P^N and $\beta_{P'}^N$ for γN scattering were determined in our previous work on nucleon Compton scattering to be

$$\begin{aligned}\beta_P^N&=0.3,\\ \beta_{P'}^N&=0.13\text{ GeV}^{-1/2}.\end{aligned} \qquad (4.7)$$

The residues $\beta_{P,P'}(\pi N)$ and $\beta_{P,P'}(NN)$ have been determined by Barger, Olsson, and Reeder[5] to be

$$\begin{aligned}\beta_P(\pi N)&=20.1\text{ mb},\\ \beta_{P'}(\pi N)&=19.8\text{ mb GeV},\\ \beta_P(NN)&=35.6\text{ mb},\\ \beta_{P'}(NN)&=44.3\text{ mb GeV}\end{aligned} \qquad (4.8)$$

in this notation. From (4.5), (4.6), (4.7), and (4.8), and using $\lambda=0.567$ GeV (see below), we obtain

$$\begin{aligned}\beta_P^\pi&=0.18,\\ \beta_{P'}^\pi&=0.15\text{ GeV}^{-1/2}.\end{aligned} \qquad (4.9)$$

The transverse cross section for γN scattering obtained in our earlier work is given in the Regge region by

$$\begin{aligned}\sigma_T(\gamma p,\gamma n)&=\frac{4\pi^2\alpha}{\nu+q^2/2M}W_1^{p,n}(\nu,q^2)\\ &=\frac{4\pi^2\alpha}{\nu+q^2/2M}\left(\frac{\omega^2-1}{\omega^2+\omega_0^2}\right)^3\\ &\quad\times\sum_i\beta_i^{p,n}\left(\frac{-q^2}{-q^2+m_0^2}\right)^{\alpha_i}\left(\frac{\omega}{2M}\right)^{\alpha_i}.\end{aligned} \qquad (4.10)$$

For sufficiently large ν, fixed q^2, we have

$$\sigma_T(\gamma p,\gamma n)=\frac{4\pi^2\alpha}{\nu}\sum_i\beta_i^{p,n}\left(\frac{\nu}{-q^2+m_0^2}\right)^{\alpha_i}. \qquad (4.11)$$

The transverse cross section for $\gamma\pi$ scattering in the same limit is

$$\sigma_T(\gamma\pi)=\frac{4\pi^2\alpha}{\nu}\sum_i\beta_i^\pi\left(\frac{\nu}{-q^2+\lambda^2}\right)^{\alpha_i}. \qquad (4.12)$$

If we assume that the factorization relation (4.5) also holds for massive photons, then for fixed q^2 and $\nu\to\infty$ we get from (4.5), (4.11), and (4.12)

$$\mu\beta_i^\pi\left[\frac{1}{2\mu(-q^2+\lambda^2)}\right]^{\alpha_i}=M\beta_i^p\left[\frac{1}{2M(-q^2+m_0^2)}\right]^{\alpha_i}\frac{\beta_i(\pi N)}{\beta_i(NN)}, \qquad (4.13)$$

for $i=P,P'$. From this result, we infer that for $\nu\to\infty$, the q^2 dependence on both sides of (4.13) is the same and therefore $\lambda=m_0$. The constant m_0 has been determined in our previous calculation of the total γN cross sections and we found that $m_0=0.567$ GeV gave a satisfactory fit.

The structure functions $\overline{W}_i^\pi$, which describe the process $e^++e^-\to\pi^\pm+$ hadrons, will be obtained by calculating the discontinuity of the amplitude $\overline{T}_i^\pi$, which is the analytic continuation of the amplitude T_i^π into the region $\nu<0$ and $q^2>0$. Provided that this continuation of the model is valid, the annihilation structure functions for $-1<\omega<2\mu/-q$ are given by

$$\nu\overline{W}_2^\pi(\omega,q^2)=-\left(\frac{\omega^2-1}{\omega^2+\eta^2}\right)^2\sum_i\beta_i^\pi\left(\frac{q^2}{q^2-m_0^2}\right)^{\alpha_i}\left(\frac{-\omega}{2\mu}\right)^{\alpha_i-1}$$

and

$$2\mu\overline{W}_1^\pi(\omega,q^2)=2\mu\left(\frac{\omega^2-1}{\omega^2+\eta^2}\right)^2\sum_i\beta_i^\pi\left(\frac{q^2}{q^2-m_0^2}\right)^{\alpha_i}\left(\frac{-\omega}{2\mu}\right)^{\alpha_i}. \qquad (4.14)$$

We choose the threshold behavior to be $(\omega-1)^p$

as $\omega \to 1$. The relation (3.11) and the positivity of W_1^π and $\bar{W}_1^\pi$ force p to be even, so that W_1^π does not change sign in going through $\omega = 1$. The choice $p = 2$ corresponds to a pion form factor which behaves as $F_\pi(q^2) \simeq (-q^2)^{-3/2}$ as $-q^2 \to \infty$, which is consistent with the presently available data.

It should be mentioned at this juncture that the model we have is *not* simply a Regge-pole model which has a scale-invariant limit. The Regge limit corresponds to $-q^2$ fixed, $\nu \to \infty$. However, the model is valid for all $|\nu| > 2$ GeV; thus, its region of validity includes, for example, the limits ν large, q^2 large but $\omega < 12$, which is certainly not characterized by Regge behavior in either ν or ω, if we consider the SLAC data.[3] For this reason, Regge behavior in ν does not necessarily occur when $\cos\theta_t \gg 1$, i.e., $\nu \gg \sqrt{-q^2}$, but rather when $\nu > 3$ GeV *and* $\omega > 10$. When we calculate the annihilation structure functions, we may have $|\cos\theta_t| \sim 1$ and the problem is how to continue the Regge part of the model. The continuation $P_\alpha(z) + P_\alpha(-z)$ is not valid for varying photon mass; in particular, it is not correct for $|q^2|$ small. Thus, we have chosen simply to use the form $(\nu + \epsilon)^\alpha + (\epsilon - \nu)^\alpha$, with ϵ small even in the region where $|\cos\theta_t| \sim 1$. In fact, $\cos\theta_t = \nu/|q| = -\omega|q|/2\mu$. Further, $-1 \leq \omega \leq -2\mu/|q|$ whereby for the annihilation region $1 \leq \cos\theta_t \leq |q|/2\mu$; so in the scale-invariant limit we do not expect the region where $\cos\theta_t \sim 1$ to be very heavily weighted in the total cross section because the cross section (3.16) is zero at $\cos\theta_t = 1$.

We note that fixed poles, for example at $J = 0$, may be present in the amplitude.

We are now in a position to calculate the total cross section for $e^+ + e^- \to \pi^\pm$ + hadrons. In Fig. 4, we show the predicted curve for $\eta^2 = 0.065$ compared with the data from the Frascati storage-ring experiment[1] and we see that our calculated cross section compares reasonably well with the rough data. However, the events corresponding to inelastic leptonic and electromagnetic processes, such as $e^+ + e^- \to \mu^+ + \mu^- +$ "third particle," have not been subtracted out of the experimental cross section, and there is some ambiguity about the discrimination against events corresponding to two-photon exchanges, which may be sizeable.[6] Furthermore, two-body final states, such as $e^+ + e^- \to \pi^+ + \pi^-$, are excluded in the experiment; these, however, amount to about 10% of the measured cross section. The quantity plotted is actually $k\sigma(4)$, where

$$k = 1 + \frac{1}{5.2}\frac{\sigma(2)}{\sigma(4)},$$

and $\sigma(n)$ is the cross section for producing n charged particles + possible neutrals. We may interpret $k\sigma(4)$ as a rough approximation to the multihadron production cross section, although it may differ from the latter considerably (~70%); therefore, we interpret our agreement with the data in a qualitative sense.

It is interesting that our predicted cross section does not fall off rapidly as a function of q^2, in agreement with the preliminary data. As $q^2 \to \infty$, the total cross section will eventually decrease as $\sigma \sim 1/q^2$, as expected on the basis of a scale-invariant model. Of course, this asymptotic behavior sets in after the scale-invariance breaking effects have disappeared in the model in the limit $q^2 \to \infty$.

We observe that for the annihilation process

$$\sqrt{q^2} \leq \nu \leq q^2/2\mu \tag{4.15}$$

or

$$-1 \leq \omega \leq -2\mu/\sqrt{q^2}\,. \tag{4.16}$$

The threshold for two-pion production is $q^2 = 4\mu^2$ so that $\nu = 2\mu = 280$ MeV, and therefore a Regge model is not expected to be valid for this process. However, the model may be expected to describe adequately production above the four-pion threshold.

V. FINITE-ENERGY SUM RULE AND FORM FACTORS

Let us now consider the finite-energy sum rule proposed by Bloom and Gilman[7] which for γp scattering takes the form

$$\frac{2M}{-q^2}\int_{-q^2/2M}^{\nu_p} d\nu\, \nu W_2^p(\nu, q^2) = \int_1^{2M\nu_p/(-q^2)} d\omega\, \nu W_2^p(\omega) \quad (-q^2 \text{ large}), \tag{5.1}$$

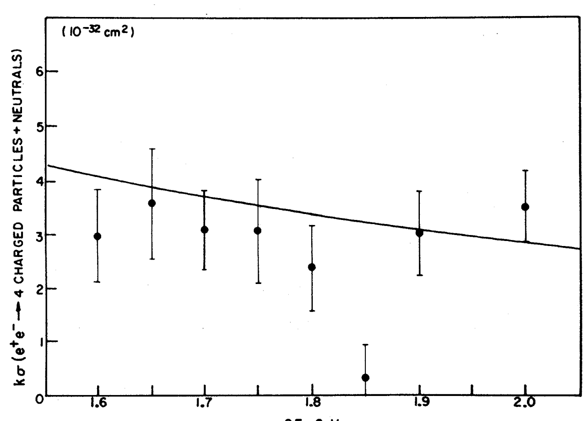

FIG. 4. Plot of $k\sigma(e^+ + e^- \to 4$ charged particles + neutrals) vs center-of-mass energy $2E_\gamma$. The solid line is our fit; the data are from Ref. 1. For an explanation of k, see text.

where on the right-hand side we have assumed that νW_2 is scale-invariant, i.e., depends only upon $\omega = -2M\nu/q^2$. For $\gamma\pi$ scattering, we have

$$\frac{2\mu}{-q^2}\int_{-q^2/2\mu}^{\nu_\pi} d\nu\, \nu W_2^\pi(\nu, q^2) = \int_1^{2\mu\nu_\pi/(-q^2)} d\omega\, \nu W_2^\pi(\omega) \quad (-q^2 \text{ large}). \quad (5.2)$$

These sum rules imply that for $\nu < \nu_{p,\pi}$, $\nu W_2(\omega)$ acts as a smooth function which averages $\nu W_2(\nu, q^2)$ in the sense of finite-energy sum rules. Bloom and Gilman[7] then extend (5.1) and (5.2) by making the very strong assumption of locality; i.e., in the vicinity of a resonance [including the nucleon and pion poles, in (5.1) and (5.2), respectively], $\nu W_2(\omega)$ is still a good average of the resonance bump which appears in $\nu W_2(\nu, q^2)$. The contribution of the nucleon form factors to νW_2^p is

$$\nu W_2^p(\text{nucleon}) = 2M\nu[G(q^2)]^2\delta(s - M^2) = -q^2[G(q^2)]^2\delta(s - M^2), \quad (5.3)$$

where

$$[G(q^2)]^2 = \frac{[G_E(q^2)]^2 + (-q^2/4M^2)[G_M(q^2)]^2}{1 + (-q^2/4M^2)}. \quad (5.4)$$

Similarly, the contribution of the pion form factor to νW_2^π is

$$\nu W_2^\pi(\text{pion}) = 2\mu\nu[F_\pi(q^2)]^2\delta(s - \mu^2) = -q^2[F_\pi(q^2)]^2\delta(s - \mu^2). \quad (5.5)$$

Thus, with the assumption of locality, the sum rules (5.1) and (5.2) become

$$[G(q^2)]^2 = \int_1^{1+(W_p^2 - M^2)/(-q^2)} \nu W_2^p(\omega)\, d\omega \quad (5.6)$$

and

$$[F_\pi(q^2)]^2 = \int_1^{1+(W_\pi^2 - \mu^2)/(-q^2)} \nu W_2^\pi(\omega)\, d\omega. \quad (5.7)$$

The upper limits are somewhat arbitrary, but in general are chosen so as to include most of the effect of the hadron pole, and not too much contribution from higher resonances. We are, after all, approximating an integral over a δ function [on the left-hand side of (5.1) and (5.2)] by an integral over a smooth function over some finite range (on the right-hand side). We shall assume that the "scaling" relations between the nucleon form factors are valid,[8] i.e.,

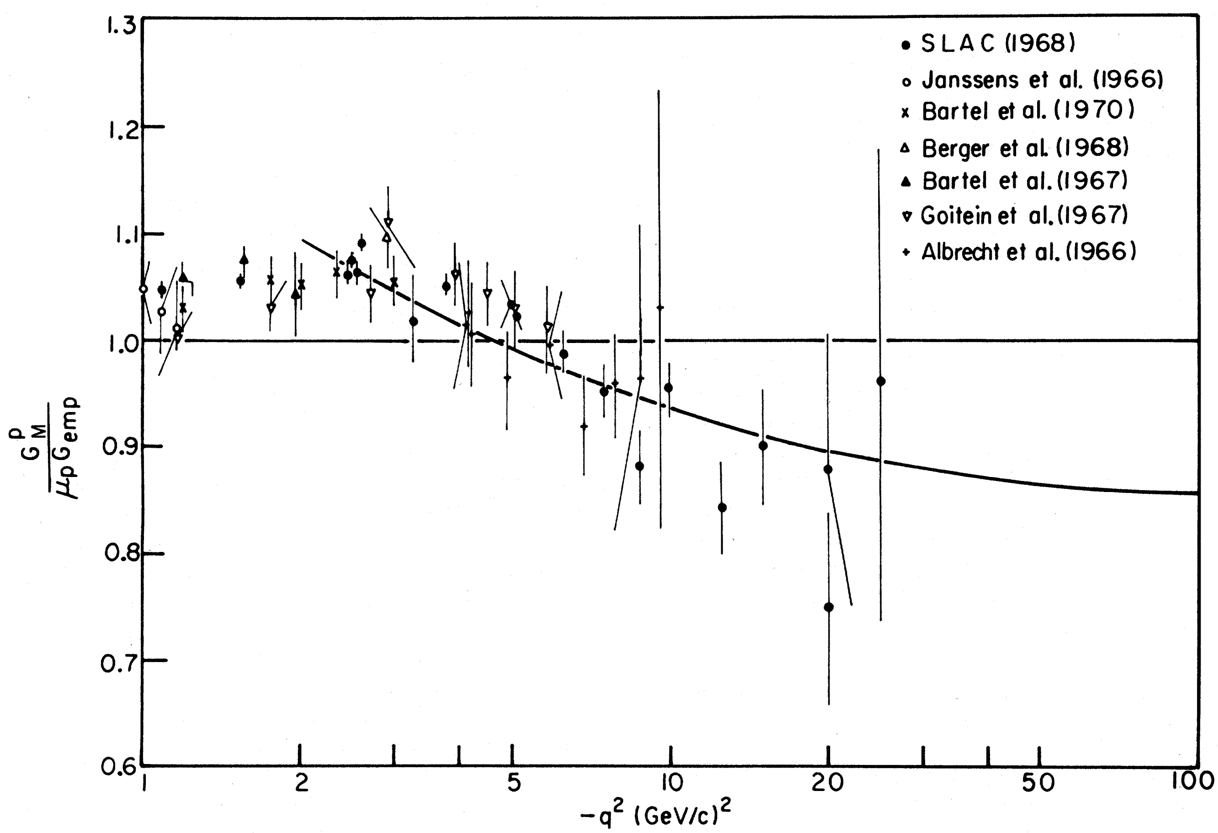

FIG. 5. Proton form factor $G_M^p(q^2)/\mu_p G_{\text{emp}}$ vs momentum transfer squared, $-q^2$. $G_{\text{emp}} \equiv 1/(1 - q^2/0.71)^2$. The solid line is our fit; the data are from Ref. 9.

$$G_E^p(q^2)=\frac{G_M^p(q^2)}{\mu_p}=\frac{G_M^n(q^2)}{\mu_n},\qquad G_E^n(q^2)=0, \tag{5.8}$$

where $\mu_p=2.79$, $\mu_n=-1.91$. If we assume that the scaling relations (5.8) hold, then (5.4) becomes

$$[G(q^2)]^2=\left(\frac{G_M(q^2)}{\mu_p}\right)^2\frac{1+(-q^2\mu_p{}^2/4M^2)}{1+(-q^2/4M^2)} \tag{5.9}$$

and in the limit $-q^2\to\infty$, it follows that $G(q^2)\approx G_M^p(q^2)$.

If we differentiate (5.7) with respect to q^2, we get

$$\frac{d}{dq^2}[F_\pi(q^2)]^2=\nu W_2^\pi\left(1+\frac{W_\pi{}^2-\mu^2}{-q^2}\right)\frac{W_\pi{}^2-\mu^2}{-q^4}, \tag{5.10}$$

$$\nu W_2^\pi\left(1+\frac{W_\pi{}^2-\mu^2}{-q^2}\right)=\frac{-q^4}{W_\pi{}^2-\mu^2}2F_\pi(q^2)F_\pi'(q^2), \tag{5.11}$$

where

$$F_\pi'(q^2)=\frac{dF_\pi(q^2)}{dq^2}.$$

In the limit $q^2\to 0$, ν fixed ($\omega\to\infty$), we find

$$\langle r_\pi\rangle=\lim_{q^2\to 0}\frac{3(W_\pi{}^2-\mu^2)}{q^4}\nu W_2^\pi(\nu,q^2). \tag{5.12}$$

Naturally, in taking this limit we must include the q^2 dependence in our model for νW_2^π in the right-hand side of the sum rule (5.7); thus in the model we are able to extrapolate to $q^2=0$ for large ν. However, we have

$$\lim_{q^2\to 0}\frac{1}{q^2}\nu W_2^\pi(\nu,q^2)\neq 0$$

and the sum rule therefore leads to the result $\langle r_\pi\rangle=\infty$. We also observe from (5.1) and (5.2) that as $q^2\to 0$, the range of integration runs over values of ν which are below the region for which the Regge-pole model is valid. Therefore, the sum rules are not reliable for small values of $-q^2$ when we describe the structure functions by our model. But we can use (5.6) and (5.7), together with (5.9), to predict the elastic form factors of the pion and the nucleon at larger values of $-q^2$. In Fig. 5, we show the predicted form-factor ratio $G_M^p/\mu_p G_{\rm emp}$, where

$$G_{\rm emp}=\frac{1}{(1-q^2/0.71)^2}, \tag{5.13}$$

and compare it to the data.[9,10] We have included the scale-invariance-breaking term in our evaluation of the right-hand side of (5.6), since our model only has exact scale invariance as $-q^2\to\infty$, $\nu\to\infty$. We have used the parameters in our model for νW_2^p derived previously,[3] with $W_p{}^2=2.2$ GeV^2. We see that the fit to the data for $-q^2\gtrsim 2$ GeV^2 is good. Of course, the $1/q^4$ behavior for large $-q^2$ is built in. The predicted pion form factor $F_\pi(q^2)$ for $W_\pi{}^2=1.29$ GeV^2 is shown in Fig. 6, and compared with the available data.[11] For the electroproduction process, we have in (5.6)

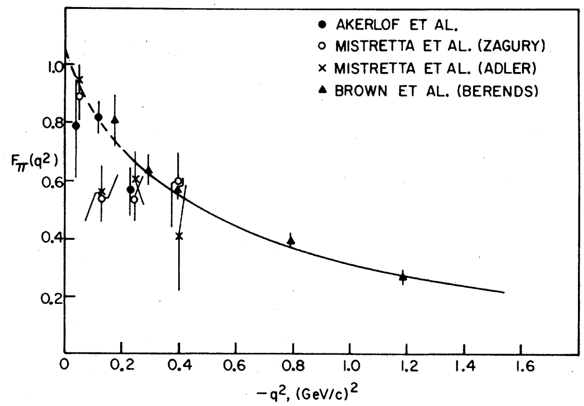

FIG. 6. Pion form factor $F_\pi(q^2)$ vs momentum transfer squared, $-q^2$. The solid line is our fit; the data are from Ref. 11.

$$1\leqslant\omega\leqslant 1+\frac{W_p{}^2-M^2}{-q^2} \tag{5.14}$$

and this gives

$$\frac{-q^2}{2M}\leqslant\nu\leqslant\frac{-q^2+W_p{}^2-M^2}{2M}. \tag{5.15}$$

Thus, a value of $-q^2=2$ GeV^2 corresponds to the energy range $1\leqslant\nu\leqslant 1.8$ GeV, wherein we expect our Regge model to be valid, albeit marginally at the lower limit. Hence, we present a fit only for $-q^2\geqslant 2$ GeV^2. For the pion form factor, the range in ν is

$$\frac{-q^2}{2\mu}\leqslant\nu\leqslant\frac{-q^2+W_\pi{}^2-\mu^2}{2\mu} \tag{5.16}$$

and for $-q^2=0.2$ GeV^2 this corresponds to the range $0.7\leqslant\nu\leqslant 4.2$ GeV. Also, in this case the Regge-pole model for νW_2^π, in conjunction with the sum rule (5.7), gives a reasonable fit to the data even in the non-Regge region (dashed line in Fig. 6).

We have shown that a slow decrease of the e^+e^- annihilation cross section is consistent with a pion form factor that falls off fairly rapidly for space like q^2, which behavior suggests that the pion is not a pointlike structure, but appears to have structure comparable to the nucleon. Therefore, one can say that a flat e^+e^- annihilation cross section is not inconsistent with a rapidly decreasing pion form factor.

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[8]Actually, a much weaker condition suffices for large q^2. We simply need

$$\frac{G_E^p(q^2)}{q^2 G_M^p(q^2)} \to 0 \quad \text{as } q^2 \to \infty$$

which is certainly borne out by presently available data.

[9]T. Janssens *et al.*, Phys. Rev. 142, 922 (1966); D. H. Coward *et al.*, Phys. Rev. Letters 20, 292 (1968); W. Bartel *et al.*, Phys. Letters 33B, 245 (1970); C. H. Berger *et al.*, *ibid.* 28B, 276 (1968); W. Bartel *et al.*, *ibid.* 25B, 236 (1967); W. Albrecht *et al.*, Phys. Rev. Letters 17, 1192 (1966); M. Goitein *et al.*, *ibid.* 18, 1016 (1967). This list is not intended to be exhaustive but represents the major amount of presently available data.

[10]This result for the nucleon form factor was reported in Ref. 3, but no detailed fit to the data was presented.

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Violation of the $|\Delta\vec{I}| = \frac{1}{2}$ Rule in the K_{l3} Decays*

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The violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule in the K_{l3} decays is discussed in detail. The effect of broken symmetry on weak hadron currents is taken into account by applying the hypothesis of asymptotic SU(3) and SU(2) symmetries. No assumption is made about the mechanism of the SU(3) and SU(2) breakings except that both of them belong to an octet. The main effect turns out to be described essentially by the effect of the particle mixings which take place among the π^0, η^0, and η'^0 (958) mesons. It is shown that one of the solutions for these mixing parameters, which is compatible with the SU(3) and SU(2) mass splittings between the members of the pseudoscalar nonet, gives rise to a rather sizable violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule in the branching ratios, and in particular, the sign of the violation is opposite to the one obtained by the usual consideration of the electromagnetic correction involving charged leptons. The solution predicts $R_e \simeq \Gamma(K_L \to \pi^\pm e^\mp \nu)/2\Gamma(K^+ \to \pi^0 e^+ \nu) \simeq 0.94$ and also $R_e \simeq R_\mu$, where R_μ is the corresponding branching ratio for the muon decays. This seems to be consistent with the latest compilation of world data. A remark is also added on the semileptonic baryon processes in broken SU(3) and SU(2) symmetries.

I. INTRODUCTION

Usually, weak semileptonic processes are assumed to take place through the weak hadronic currents which obey the $|\Delta\vec{I}|=\frac{1}{2}$ rule. However, in the presence of an SU(2)-breaking interaction this selection rule is an approximate one. The electromagnetic interaction need not be the only cause of this violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule. There is some room left for suspecting the existence of a nonelectromagnetic isospin-violating interaction.

One of the most promising places to study the violation of the $|\Delta\vec{I}|=\frac{1}{2}$ rule is the comparison of the K_{l3}^+ and K_{Ll3}^0 decays. Starting from a phenomenological local weak interaction of the form

$$L \propto [(p_K + p_\pi)_\alpha f_+(q^2) + (p_K - p_\pi)_\alpha f_-(q^2)]\,\bar{u}_\nu \gamma_\alpha (1+\gamma_5) u_l \tag{1}$$